9.4: Equations Reducible to Quadratic

In this lesson, we will see how to take an equation that is not quadratic and apply quadratic methods to solve it.

Quadratic Form

An equation is in **quadratic form** if it can be written as

$$a(u)^2 + b(u) + c = 0.$$

If an expression is a trinomial, we can recognize that it is in quadratic form if the power on one variable (or expression)term is *twice as large* as the next term's power.

Step 1: Set the equation equal to 0
Step 2: Determine if it is in quadratic form and define the value of u
Step 3: Substitute u into the equation and use quadratic methods to solve for u
Step 4: Re-substitute the original expression for u and solve for original variable.
Step 5: Check answer in original equation.

Try it Determine if the equations are in quadratic form, then define the value of u.

a) $x^4 - 5x^2 + 6 = 0$

b) $(x^2 - 3)^2 + (x^2 - 3) - 5 = 0$

c) $x + 8\sqrt{x} - 9 = 0$

d) $2\,m^{-2} - 5\,m^{-1} - 12 = 0$

e) $t^{\frac{2}{3}} - t^{\frac{1}{3}} - 2 = 0$

f) $\left(10 - \sqrt{x}\right)^2 - 2\left(10 - \sqrt{x}\right) - 35 = 0$

Substitute and Solve

Now let's go the next step and solve these equations using u-substitution.

a) $x^4 - 5x^2 + 6 = 0$

b) $(x^2 - 3)^2 + (x^2 - 3) - 5 = 0$

c) $x + 8\sqrt{x} - 9 = 0$

d) $2\,m^{-2} - 5\,m^{-1} - 12 = 0$

Example **Find the x –intercepts of these functions**

a) $g(t) = t^{\frac{2}{3}} - t^{\frac{1}{3}} - 2$

b) $h(x) = \left(10 - \sqrt{x}\right)^2 - 2\left(10 - \sqrt{x}\right) - 35$

9.5: Applications of Quadratic Equations

In this lesson, we will explore some real-world problems that involve quadratic equations.

Area Problems

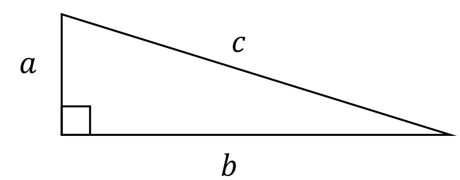
Start with a Triangle

An architect is designing the entryway of a restaurant. She wants to put a triangular window above the doorway. Due to energy restrictions, the window can only have an area of 120 square feet and the architect wants the base to be 4 feet more than twice the height. Find the base and height of the window.

Pythagorean Theorem

In this lesson we will investigate several applications of radical expressions and equations beginning with one of the most important equations in math, the Pythagorean Theorem.

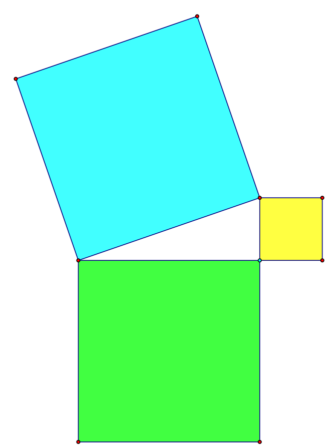
I. **Right Triangles and The Pythagorean Theorem**



Pythagorean Theorem: *"In all right triangles,*
The sum of the area of the squares on the legs is equal to the area of the square on the hypotenuse."

Algebraically: *If a triangle is a right triangle with legs of length a and b, and a hypotenuse of length c, then…*

$$a^2 + b^2 = c^2$$



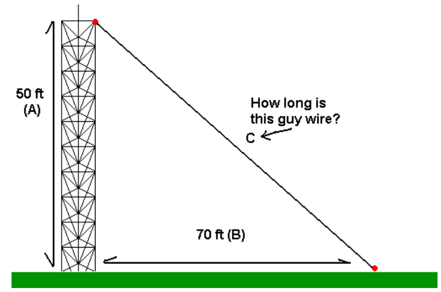
Example 1 Find the missing side for the following right triangles. State your answer in simplest radical form.

a) $a = 10, c = 26$

b) The shortest leg of a right triangle is 2 cm shorter than the other leg. The hypotenuse is 10. What are the lengths of the two legs of the triangle?

Example 2

a) Suppose a 50 foot **cell tower** is being built, and a guy wire is going to be attached to the ground 70 feet from the base. How long will this wire need to be?



b) Suppose we have a 100 foot **wire** that we want to use, how far from the base should the anchor be put in the ground to attach this 100 ft. guy wire?

Falling Objects

The distance that a free-falling object travels is $s(t) = 16t^2$. If you drop a penny from the top of the roof of the Empire state building at 1250 feet high, how long will it take to reach the ground?

Projectiles

The height of a projectile (an object that is propelled into the air without any means of self-propulsion) is determined by the function

$$s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$$

Where

$$s_0 = intial\ height,\ \ v_0 = initial\ vertical\ velocity$$

$$g = acceleration\ of\ gravity = 32 \frac{ft}{sec.^2} \approx 9.8 \frac{m}{sec.^2}$$

Note that this only measures vertical height and disregards the horizontal position.

Example
A bullet is shot in the air off of a 128 foot building with an initial vertical velocity of 320 ft. per second.
The function for the height of the projectile is $s(t) = 128 + 320t - 4.9t^2$

a. What will the height of the projectile be after 1 second?

b. Find out how long it will take the bullet to hit the ground.

Predicting Weather:
The amount of precipitation in Sonoma, California, can be approximated by
$$P(x) = 0.2x^2 - 2.8x + 9.8$$
where $P(x)$ is in inches and x is the number of the month ($x = 1$ corresponds to January, etc.).

What is the amount of precipitation predicted in March?

When will there be no precipitation?

<u>Example</u> (If time permits) **Business Model:**
When computing revenue (the amount of money that a business brings in) we often get a quadratic model based on a linear relationship between price and the number of units sold.

Suppose that the *Swedish Sister's* coffee stand would "sell" 2400 Lattes in a week if they gave them away for free (because that's all they can make). After collecting some data over several months, their marketing department has found that they sell 400 fewer Lattes for every dollar that they raise the price.

a) Write an equation for the number of lattes, n, as a function of the price p in dollars.

b) At what price will $n = 0$? Explain what this means for the Swedish Sisters?

c) Revenue is calculated by multiplying the number of lates sold, n, by the price p.
$$R = p \cdot n$$
Write an equation for revenue as a function of the price p by substituting your equation from part (a) for n.

d) According to this model, what is the maximum revenue that the coffee stand will make?

e) At what price will they reach this maximum?